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Heat transfer analysis of a non-Newtonian fluid on a power-law stretched surface with suction or injection for uniform and cooled surface temperature

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Abstract *Heat transfer characteristics of a non-Newtonian fluid on a power-law stretched surface with suction or injection were investigated. Similarity solutions of the laminar boundary layer equations describing heat transfer flow in a quiescent fluid were obtained and solved numerically. Temperature profiles as well as the Nusselt number* Nu, *were obtained for two thermal boundary conditions; namely, uniform surface temperature* (b = 0) *and cooled surface temperature* (b = –1), *for different governing parameters such as Prandtl number* Pr, *injection parameter* d *and power-law index* n. *It was found that decreasing injection parameter* d *and power-law index* n *and increasing Prandtl number* Pr *enhanced the heat transfer coefficient.*

Nomenclature

b = temperature exponent parameter
C = dimensional constant $[K.m^{-b}]$
d = dimensionless injection parameter $[v_w/(U_o x^p)]$
$\boldsymbol{f}$ = dimensionless stream function
h = heat transfer coefficient $[W/(m^2.K)]$
k = thermal conductivity $[W/(m.K)]$
n = fluid power-law index
Nu = Nusselt number $[hx/k]$
p = velocity exponent parameter
Pr = generalized Prandtl number $[v^*(U_o x^p)^{n-1} x^{1-n}/\alpha]$
Re = generalized Reynolds number $[(U_o x^p)^{2-n} x^n/v^*]$
T = temperature $[K]$
U_o = dimensional constant $[m^{1-p}/s]$
u = velocity component in the x-direction $[m/s]$
v = velocity component in the y-direction $[m/s]$
x = coordinate in direction of surface motion $[m]$
y = coordinate in direction normal to surface motion $[m]$

Greek symbols

α = thermal diffusivity $[m^2/s]$
η = dimensionless similarity variable $[y/x]$
θ = dimensionless temperature $[(T - T_\infty)/(T_w - T_\infty)]$; $[(T - T_\infty)/(Cx^b)]$
μ = dynamic viscosity$[kg/(m.s)]$
μ^* = generalized dynamic viscosity $[kg/m.s^{2-n})]$
ν^* = generalized kinematic viscosity $[m^2/s^{2-n}]$
ρ = density $[kg/m^3]$

Subscripts

w = condition at the surface
x = derivative with respect to x
∞ = condition at ambient medium

Introduction

Thermal transport from a heated moving surface to a quiescent non-Newtonian fluid is of interest in many practical industrially important processes such as multiphase mixtures, polymer melts and solutions, food products, biological fluids, natural products and agricultural and dairy wastes. The interest in studying flow and heat transfer characteristics of non-Newtonian fluids has increased in the last four decades because of their important usage and wide range of applications. Considerable efforts have been directed at these characteristics to control the quality of the final product of these processes because of the growing use of these fluids in various manufacturing and processing industries such as hot rolling, extrusion, wire drawing, continuous casting, glass fibre production, and paper production (Chabra, 1993; Altan *et al.*, 1979; Fisherr, 1976).

Many authors have attacked the problem from the point of view of a plate moving with a linear velocity and for various temperature boundary conditions. Grubka and Bobba (1985) studied the heat transfer characteristics of a continuous stretching surface with variable temperature. Furthermore, linearly stretching surface subject to suction or injection was studied. Chen and Char (1988) studied the effects of power-law surface temperature and power-law surface heat flux variation on the heat transfer characteristics of a continuous, linearly stretching sheet subject to blowing or suction in a fluid initially at rest and at uniform temperature. Vajravelu and Rollins (1991) investigated the heat transfer characteristics in a visco-elastic fluid over a continuous, impermeable, linearly stretching sheet with power-law surface temperature or power-law surface heat flux. Ahmad and Mubeen (1995) investigated the heat transfer characteristics for large and small Prandtl numbers in a boundary layer flow of incompressible viscous fluid past a stretching plate with suction.

Ali (1995) studied the similarity solutions of the laminar boundary layer equations describing heat and flow in a quiescent Newtonian fluid driven by a stretched surface subject to suction or injection; he investigated, for three thermal boundary conditions, the effects of Prandtl number, temperature exponent, velocity exponent and the injection parameter. Char and Chen (1988) studied the temperature distribution in a visco-elastic fluid of Walter's liquid B model over a horizontal stretching plate. The velocity of the plate is proportional to the distance from the slit and the plate is subject to a power-law variable heat flux. Gorla *et al.* (1995) studied the free convection boundary layer equations of the Ostwald-de Waele non-Newtonian power-law type fluids near a three-dimensional stagnation point of attachment on an isothermal surface for variable, power-law index and Prandtl number. Acrivos *et al.* (1960) explored the functional relationship between characteristic groups of non-Newtonian fluids past an external surface for very large Prandtl numbers. Gorla *et al.* (1998) presented the effect of viscosity index on the surface heat transfer rate of a non-similar boundary layer analysis for the problem of mixed convection in power-law type non-Newtonian fluids along horizontal surfaces with variable wall temperature distribution.

In the present work, the laminar boundary layer flow and heat transfer characteristics of non-Newtonian fluids over a continuous stretched surface employing the most general power-law for velocity and temperature distributions with various injection parameters will be studied. Similar solutions for the governing boundary layer equations will be obtained and the derived ordinary differential equations are then solved numerically for different parameters: the power-law index n; the temperature exponent b; the injection parameter d; and the Prandtl number Pr.

Mathematical formulation

Consider a power-law stretched surface with suction or injection moving through a quiescent non-Newtonian fluid, as shown in the schematic diagram Figure 1. The x coordinate is measured along the moving surface from the point where the surface originates and the y coordinate is measured normal to it. It should be noted that positive and negative m indicates that the surface is accelerated or decelerated from the extruded slit, respectively. Positive and negative d implies injection and suction, respectively. The governing equations for steady, laminar, two-dimensional, incompressible viscous flow of a non-Newtonian fluid with constant physical properties can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial \tau_{yx}}{\partial y}$$

where

$$\tau_{yx} = \mu^* \left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}$$

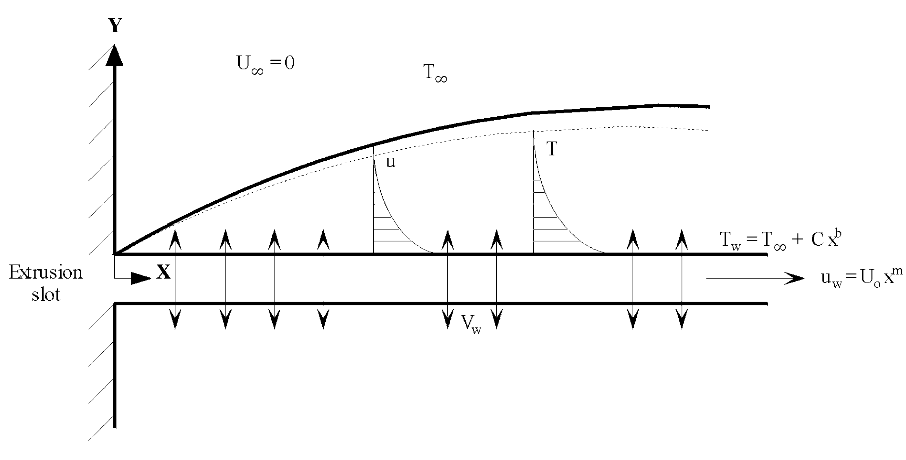

Figure 1. Schematic diagram of flow induced by a power-law stretched surface

or,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu^*}{\rho}\left[(n-1)\left|\frac{\partial u}{\partial y}\right|^{n-2}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+\left|\frac{\partial u}{\partial y}\right|^{n-1}\left|\frac{\partial^2 u}{\partial y^2}\right|\right. \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

With the following boundary conditions:

$$\left.\begin{array}{ll} @y=0 & u=U_o x^m, v=v_w(x), T=T_\infty+Cx^b \\ @y\rightarrow\infty & u\rightarrow 0 \quad , \quad T\rightarrow T_\infty \end{array}\right\} \tag{4}$$

two thermal boundary conditions of uniform temperature ($b=0$) and cooled surface temperature ($b=-1$) are considered.

Problem solution

The governing equations (1-3) are very complex because of their high non-linearity; therefore a similarity transformation is introduced in order to facilitate the solution. The transformed equations are solved numerically using a finite difference method. The stream function ψ is defined so that it satisfies the continuity equation:

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x} \tag{5}$$

The similarity variable η and the stream function ψ are defined as follows:

$$\eta(x,y)=y/x, \psi=U_o x^{\frac{2(n-1)}{n-2}} f(\eta) \tag{6}$$

The velocity components in x and y direction can be written as:

$$u=U_o x^{\frac{n}{n-2}} f'(\eta), v=U_o x^{\frac{n}{n-2}}[\frac{2(1-n)}{n-2}f(\eta)+\eta f'(\eta)] \tag{7}$$

The temperature T is also generalized for the similarity solution, so that the generalized temperature θ is a function of η alone:

$$\theta=\frac{T-T_\infty}{T_w-T_\infty} \text{ or } \theta=\frac{T-T_\infty}{Cx^b} \tag{8}$$

The system of the governing equations is transformed into the following ordinary differential equations:

$$(q+r)f'^2-rff''=|f''|^{n-2}f'''[|f''|+(n-1)f''] \tag{9}$$

$$\theta''=\Pr(b\theta f'-rf\theta') \tag{10}$$

With their boundary conditions:

$$\left.\begin{array}{l} f'(0) = 1,\ f(0) = d_{\frac{n-2}{2\,(1-n)}},\ \ f'(\eta_\infty) = 0 \\ \theta\,(\eta_\infty) = 0, \quad \theta\,(0) = 1 \end{array}\right\} \tag{11}$$

where $q = -1, r = p + 1$ and $p = \dfrac{n}{n-2}$

the heat transfer characteristics will be studied for two thermal boundary conditions:

(1) Uniform temperature (b = 0)

$$\theta(0) = 1, C = T_w - T_\infty \tag{12}$$

(2) Cooled surface temperature

$$\theta(0) = 1, C = \frac{T_w - T_\infty}{x^b} \tag{13}$$

The local heat transfer coefficient can be expressed in dimensionless form of Nusselt number for variable and uniform surface temperature:

$$Nu = -\theta'(0) \tag{14}$$

Owing to the assumption of constant physical properties, the hydrodynamics of the flow are independent of both temperature and time. The energy equation is uncoupled with the momentum equation. Therefore the momentum equation can be solved and then, once we have the velocity field, the energy equation can be solved to obtain the temperature field. The governing equations were solved using 4th-5th Runge-Kutta method with uniform grid points. To study the effect of grid refinements, results for Nusselt number *Nu* for different injection parameter d at $n = 0.2$, $b = 0$ and $Pr = 0.72$ were obtained with doubling the node points. The change in the *Nu* variation was found to be less than 1 per cent.

Results and discussion

Temperature distributions considering different parameters were obtained for values of the power-law index, n, ranges from 0.2 to 1.4 and injection parameter d from –10 to 1.25 as well as different Prandtl numbers.

The dimensionless temperature distribution with cooled surface temperature, $b = -1$, at $Pr = 10$ and $n = 0.2$, as a function of the similarity variable for different values of dimensionless parameter d, is presented in Figure 2. Heat is transferred to the moving surface for the injection ($d > 0$) and from the surface for suction ($d < 0$) processes. As might be expected, suction thins the thermal boundary layer whereas injection thickens it.

Figure 3 shows the temperature profiles for uniform surface temperature ($b = 0$). In this Figure, the thinning of the boundary layer with injection is evident for increasing the surface temperature from a uniform to a linear relation. Also, it can be seen that all heat was transferred from the surface to the

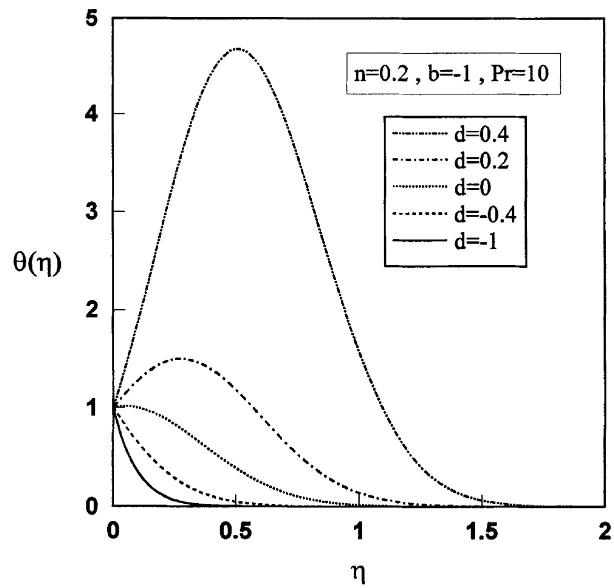

Figure 2.
Temperature profiles for cooled surface temperature at different values of dimensionless injection parameter, *d*, $n = 0.2$

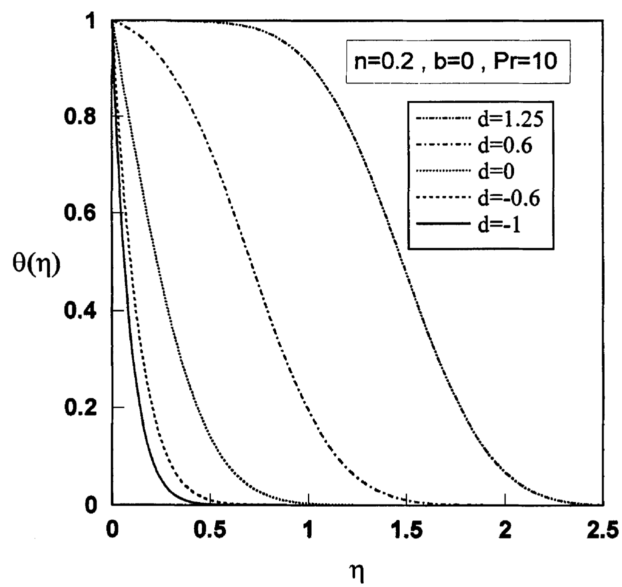

Figure 3.
Temperature profiles for uniform surface temperature at different values of dimensionless injection parameter, *d*, $n = 0.2$

medium. It should be noted that in this case the boundary layer assumptions do not permit a solution of the boundary layer equation for large d, because θ will approach a constant value of 1 and the boundary layer is almost literally blown off the surface similar to that of stationary plate with injection.

Figure 4 shows the dimensionless temperature distribution for power-law index $n = 0.4$, suction parameter, $d = -0.4$ and temperature exponent ($b = -1$) for

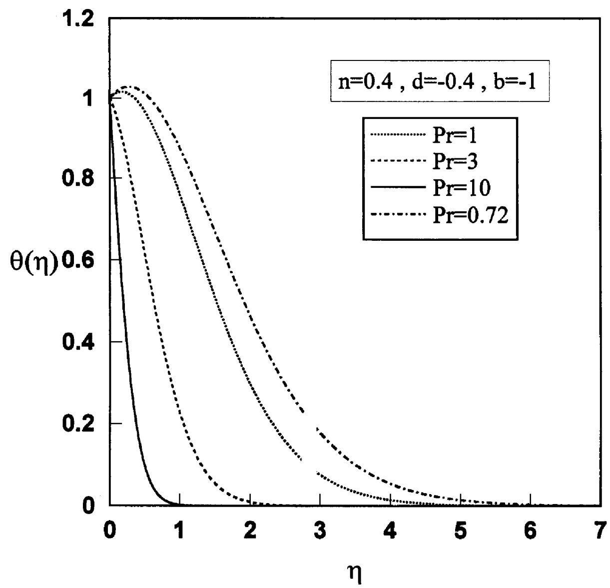

Figure 4. Temperature profiles for cooled surface temperature at different values of Prandtl number, *Pr*, $n = 0.4$

different Prandtl numbers. Increasing Prandtl number will increase adverse heat transfer near the surface and decrease the temperature as $\eta \rightarrow \eta_{\infty}$, as a result of decreasing thermal boundary layer thickness. The temperature distribution for uniform surface temperature as shown in Figure 5 is similar to the case of cooled surface temperature. Increasing Prandtl number decreases the temperature, the boundary layer thickness, and the heat is transferred from the surface to the medium. Figures 6 and 7 represent the temperature

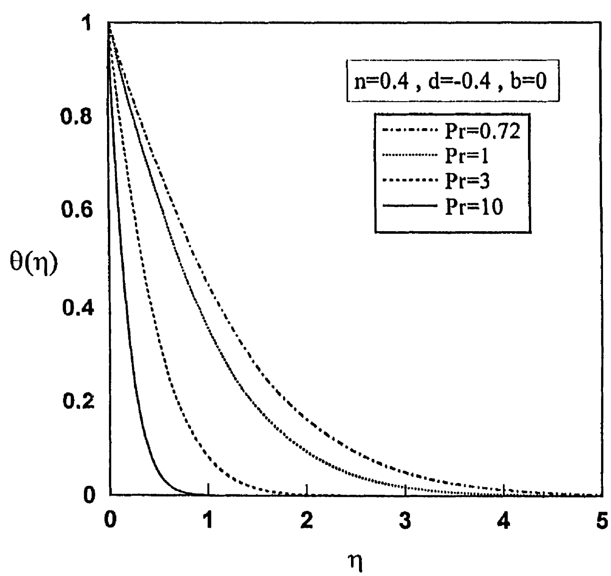

Figure 5. Temperature profiles for uniform surface temperature at different values of Prandtl number *Pr*, $n = 0.4$

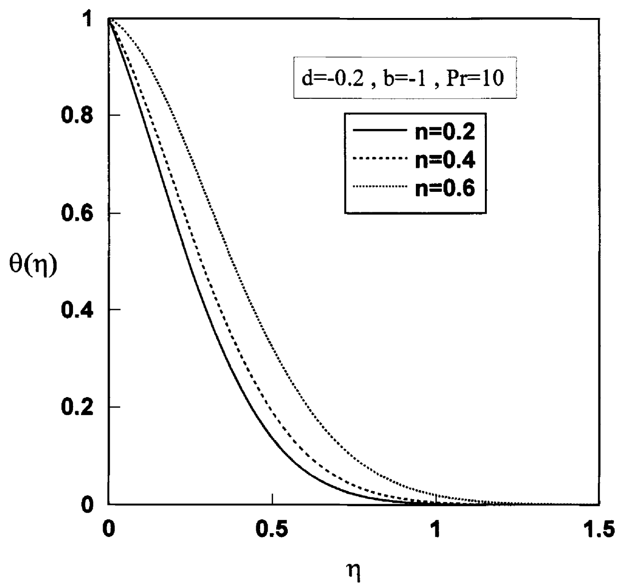

Figure 6.
Temperature profiles for cooled surface temperature at different values of flow index n

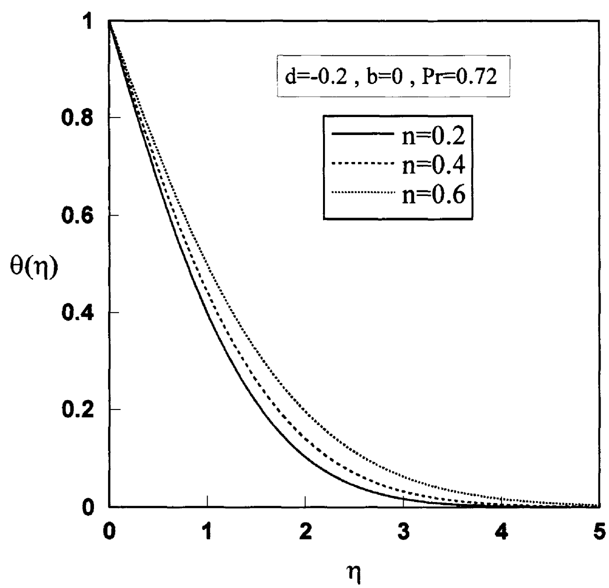

Figure 7.
Temperature profiles for uniform surface temperature at different values of flow index n

distribution at $Pr = 0.72$, $d = -0.2$ for cooled and uniform surface temperature, respectively, for different power-law index, n. Increasing power-law index will increase the temperature as a result of increasing the thermal boundary layer.

The Nusselt number Nu, as a function of the blowing parameter d, is shown in Figures 8 and 9 at $n = 0.4$ for cooled and uniform temperature distribution, respectively. Negative values of Nu indicate that heat flows into the surface

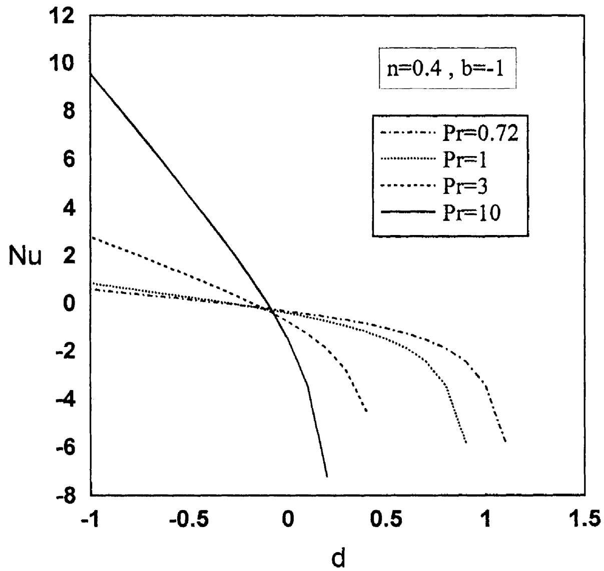

Figure 8.
Nusselt number versus dimensionless injection parameter for cooled surface temperature at different values of Prandtl number, *Pr*

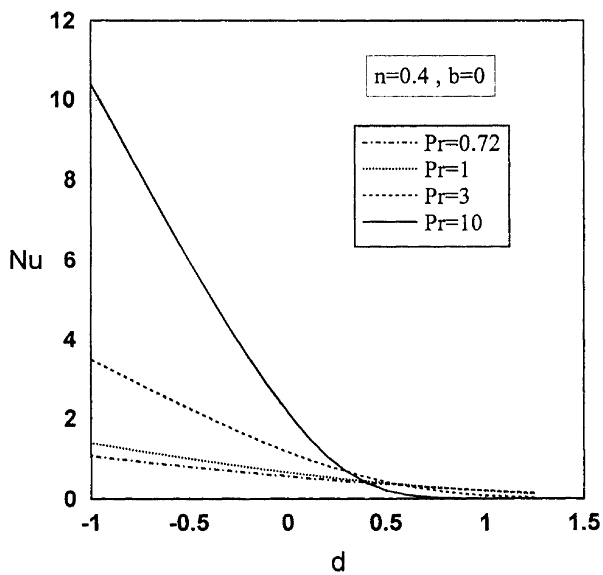

Figure 9.
Nusselt number versus dimensionless injection parameter for uniform surface temperature at different values of Prandtl number, *Pr*

despite the surface temperature's continuous excess over the free-stream temperature. This physical mechanism could be explained as a fluid particle heated to near the surface temperature, being conveyed downstream to a place at which the surface temperature is lower. Then heat flows into the surface and results in negative heat transfer coefficients, which means only that $\theta'(0)$ is no longer proportional to the temperature difference. However, positive values of *Nu* show that heat transferred from the surface to the medium results in positive heat transfer coefficient. In the case of cooled surface temperature, increasing Prandtl number will increase adverse heat transfer for injection ($d > 0$) and increase heat transfer for suction ($d < 0$).

Figure 10 shows the local heat transfer coefficient Nu as a function of d for uniform and cooled surface temperature. Increasing surface temperature b will increase heat transfer coefficient, as a result of decreasing the thermal boundary layer thickness. Figures 11 and 12 show the local Nusselt number Nu as a function of the blowing parameter d, for different power-law index n, at different values of Pr for the case of cooled and uniform surface temperature, respectively. It can be seen that increasing the power-law index n will lead to a reduction in Nusselt number Nu. This can be explained by the increase in the thermal boundary layer thickness and, as a result, the increase in the thermal resistance of this layer. Furthermore, it is clear that suction $d < 0$ enhances the heat transfer coefficient much better than blowing $d > 0$, and the thickness of the thermal boundary layer is reduced. Thus suction can be used for cooling the surface much faster than blowing.

Conclusions

In this study, heat transfer analysis of non-Newtonian fluids on a power-law stretched surface with suction or injection has been presented. Numerical solution was obtained for temperature and heat transfer coefficients for two thermal boundary conditions, uniform and cooled surface temperature. It was found that increasing the injection parameter or the power-law index increases the temperature, while increasing the Prandtl number or the surface temperature exponent decreases the temperature. In addition, suction increases the heat transfer, whereas injection decreases it for all studied parameters.

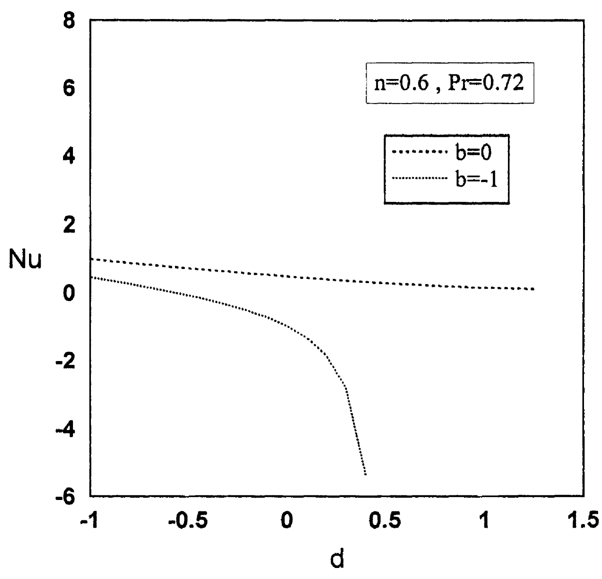

Figure 10.
Nusselt number versus dimensionless injection parameter for cooled and uniform surface temperatures

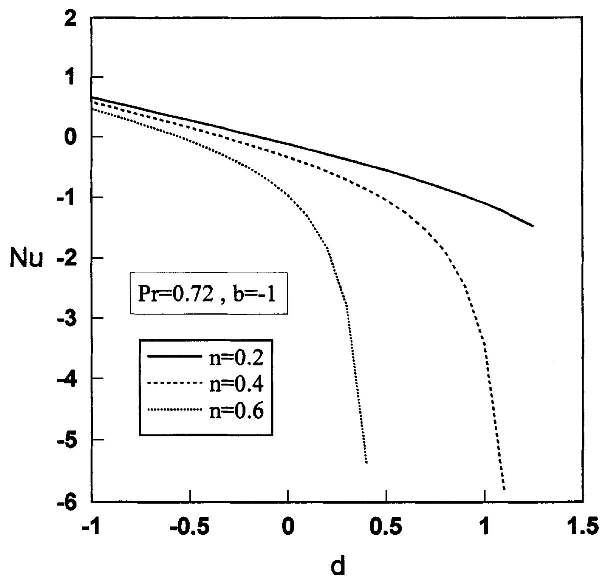

Figure 11.
Nusselt number versus dimensionless injection parameter for cooled surface temperature at different values of flow index, *n*

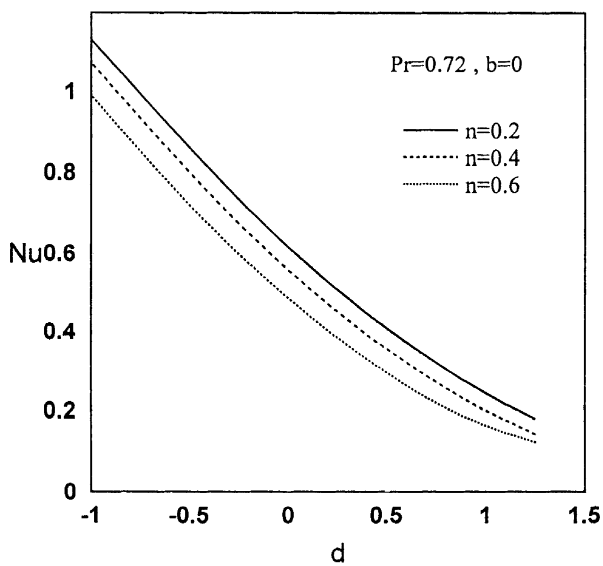

Figure 12.
Nusselt number versus dimensionless injection parameter for uniform surface temperature at different values of flow index, *n*

In the case of cooled surface temperature heat flows to or from the surface depending on d and Pr, and for all $d > 0$ negative heat transfer was found. It was found that heat transfer coefficient increases with decreasing injection parameter, and with the increase of velocity and temperature exponents at constant Pr number. Increasing Prandtl number, keeping all other parameters constant, enhances the heat transfer coefficient and increasing the power-law index will decrease the Nusselt number Nu, for all studied parameters.

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